A construction of complete-simple distributive lattices

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Abstract

In this note we prove that there exist *complete-simple distributive lattices*, that is, complete distributive lattices in which there are only two complete congruences.

1 Introduction

In this note we prove the following result:

Theorem 1 *There exists an infinite complete distributive lattice K with only the two trivial complete congruence relations.*

2 The construction

The following construction is crucial in our proof of our Theorem:

Definition 1 *Let , i∈I, be complete distributive lattices satisfying condition* (J). *Their product is defined as follows:*

that is, is with a new unit element.

Notation 1 *If i∈I and , then*

is the element of whose ith component is d and all the other components are 0.

See also Ernest T. Moynahan [].

Next we verify the following result:

Theorem 2 *Let , i∈I, be complete distributive lattices satisfying condition* (J). *Let τ be a complete congruence relation on . If there exist i∈I and with such that for all ,*

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(1)

then τ=ι.

Proof. Since

(2)

and τ is a complete congruence relation, it follows from condition (C) that

(3)

Let $j \in I$, $j \neq i$, and let . Meeting both sides of the congruence (2) with , we obtain

Using the completeness of τ and (Error: Reference source not found), we get:

hence τ=ι.

References

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